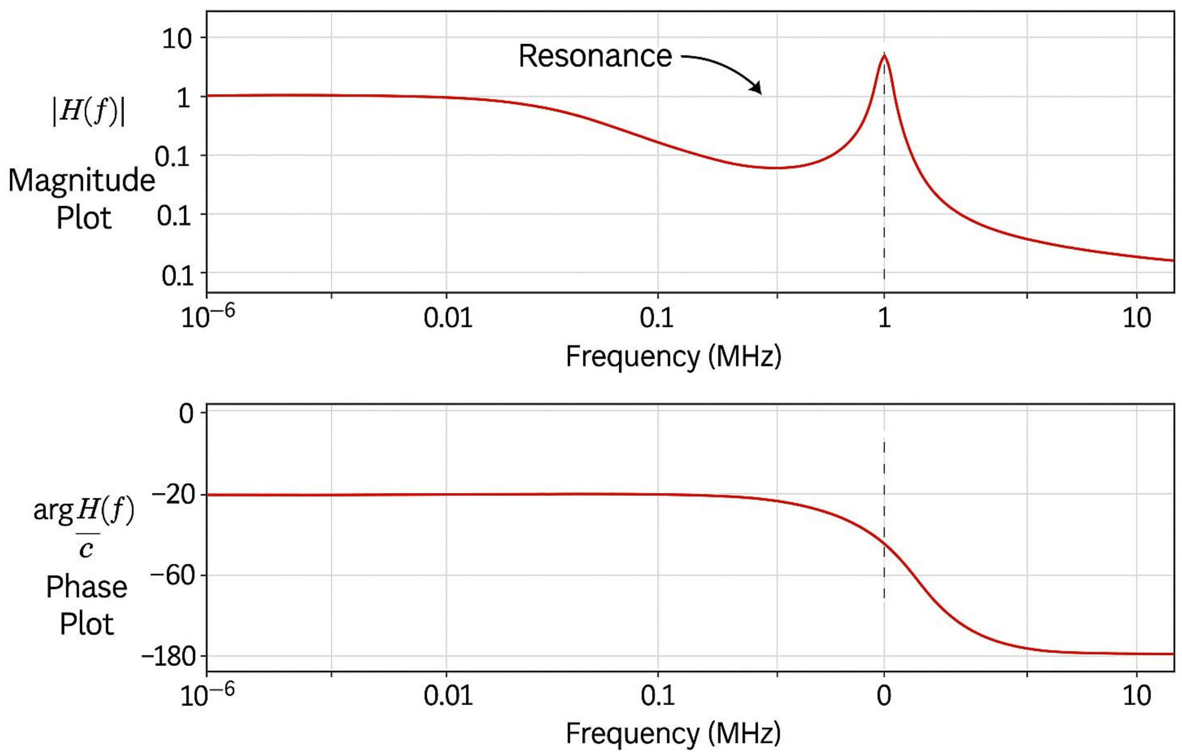


Examples Analysis Tools

A Primer on Digital Control for Power Converters

Topics and Tools

- **Review – more Like a Reminder that tools Exist**
- **LPF Example**
- **Cookbook Process**
- **Summary**
- **Construct Method**
- **HPF using Construct Method**
- **Type II using Construct Method**
- **IIR vs. FIR**
- **Shortcut LPF**
- **Convergence**
- **Tour of Supporting Files**



A Primer on Digital Control for Power Converters
Tools

Tools for the Journey

- **Euler's Formula**

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

The cornerstone of Theorems supporting signal processing and analysis.

It is important to understand that $e^{j\theta}$ represents a complex number or Phasor with a magnitude of 1.

- **Laplace Transform and inverse transform**

$$\boldsymbol{V}(\boldsymbol{s}) = \int_0^\infty e^{-st}\, v(t)dt \qquad v(t) = \frac{1}{2\pi j}\int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} e^{st}\boldsymbol{V}(\boldsymbol{s})ds$$

- **Lookup Tables and Software is available to perform most operations.**
- **We use this to ‘Jump’ between time and frequency domains**

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Tools

Review

- **Laplace Transform Pairs**
  - **A handful a transform pairs allows for most control solutions.**

Description	$f(t)$	$F(s)$
Step	$1\,u(t)$	$\frac{1}{s}$
Decay	$e^{-at}u(t)$	$\frac{1}{s+a}$
Charge	$(1-e^{-at})u(t)$	$\frac{a}{s\,(s+a)}$
Frequency Shift	$e^{at}\,f(t)u(t)$	$F(s-a)$
Time shift	$f(t-a)u(t-a)$	$e^{-as}\,F(s)$
Convolution	$\int_0^t h(\tau)f(t-\tau)d\tau$	$\mathrm{H}(s)F(s)$
Impulse	$\delta(t-t_0)$	$e^{-s\,t_0}$

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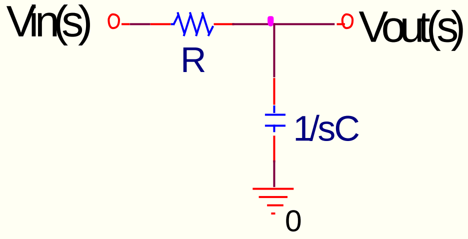
Low Pass Filter Example

Review

LPF Example



$$\frac{Vo(s)-Vi(s)}{R}+\frac{Vo(s)}{\frac{1}{sC}}=0$$

$$H(s)=\frac{G(s)}{F(s)}=\frac{Vo(s)}{Vi(s)}=\frac{\alpha}{s+\alpha}\ :\ \alpha=\frac{1}{RC}$$

As s approaches 0, H(s) approaches 1

As s approaches ∞, H(s) approaches $\frac{a}{s}$

There is a "Breakpoint" at $|s|=\alpha$

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Low Pass Filter Example

Review

LPF If we choose to work in the time domain to continue analysis, the Laplace transform table can be used Switch to the time domain.

$h(t) = \alpha\, e^{-\alpha t}$ This result is not as intuitively apparent as it was in the frequency domain.

In order to determine the LPF output in the time domain it is required to use convolution $\mathrm{g}(\mathrm{t}) = \int_0^{\mathrm{t}} \mathrm{h}(\tau)\mathrm{f}(\mathrm{t}-\tau)\mathrm{d}\tau$.

Instead of using integration, It is better to stay in the frequency domain. The problem simplifies to multiplication.

$\mathrm{G}(\mathrm{s}) = H(\mathrm{s})\mathrm{F}(\mathrm{s})$

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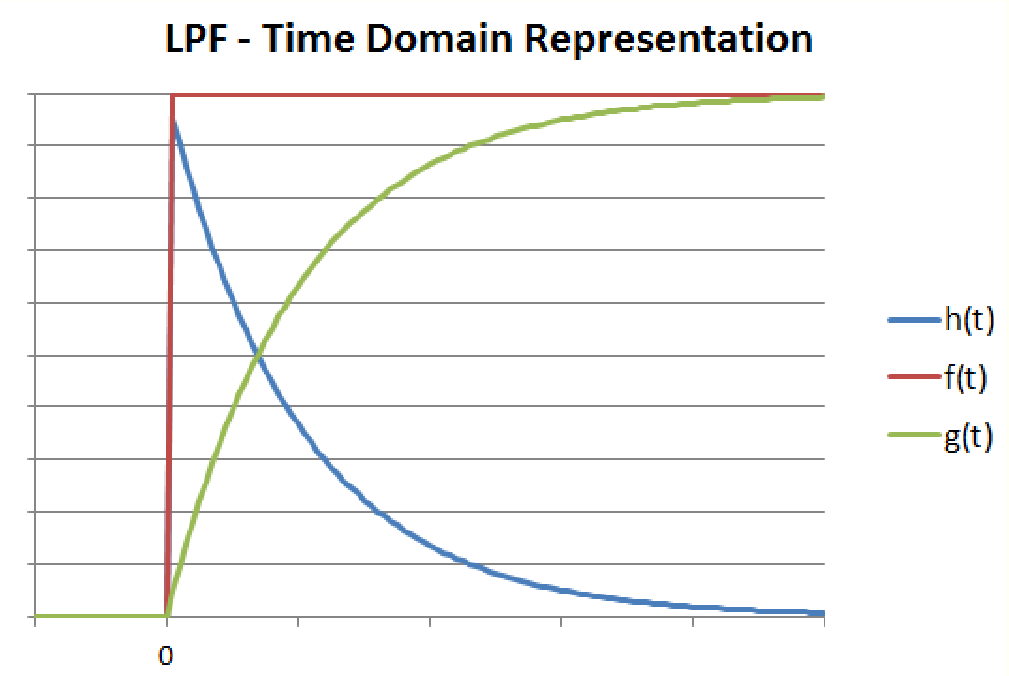
Low Pass Filter Example

Review

LPF If Applying a step function to the input of the LPF

$$G(s) = H(s)F(s) = \frac{\alpha}{s+\alpha}\frac{1}{s}$$

Using the transform table $g(t) = (1 - e^{-\alpha t})u(t)$



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Z Transform

Review

Z Transform – Maps discrete time domain to discrete frequency domain.

$$X(z) = \sum_{n=0}^{\infty} x[n]z^{-n} \qquad x[n] = \frac{1}{j2\pi}\int X(z)z^{n-1}dz$$

	Time Domain	z-Domain
Step	$u[n]$	$\frac{1}{1-z^{-1}}$
Decay	$e^{-anT}u[n]$	$\frac{1}{1-e^{-aT}z^{-1}}$
Approach	$(1-e^{-anT})u[n]$	$\frac{1-e^{-aT}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
Time Shift	$x[n-k]$	$z^{-k}X(z)$
Convolution	$h[n] * f[n]$	$H(z)F(z)$
Impulse	$\delta[n-n_0]$	$e^{-j\alpha n_o}$

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Cookbook Process

- **Start by studying an example for a LFP with a 10 kHz Breakpoint.**
- **Step 1 - Start with the frequency domain Representation**

$$\boldsymbol{H(s) = \alpha \frac{1}{\alpha + s}} \qquad \textbf{where} \qquad \alpha = 2\pi f_b \qquad \textbf{and} \qquad f_b = 10\text{kHz}$$

Step 2 Using Tables, MathCad, or by inspection Translate to the Time Domain

- **Tables…**

$$h(t) = \alpha \cdot e^{-\alpha \cdot t} \cdot u(t)$$

- **3rd Step – Translate to the Discrete time domain – replace t with n * T_m, and scale by T_m.**

$$h(n) = T_m \cdot \alpha \cdot e^{-n \cdot \alpha \cdot T_m} \cdot u(n \cdot T_m)$$

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Cookbook Process

- **Step 4 – Translate to z domain using Tables, inspection or MathCad.**

$$H(z) = T_m \cdot \alpha \cdot \frac{1}{1 - e^{-\alpha \cdot T_m} \cdot z^{-1}}$$

- **Step 5 – Format the results in normal recursive form – Denominator has a leading 1!!!**

$$H(z) = \frac{b_0 + b_1 \cdot z^{-1} + \cdots + b_n \cdot z^{-n}}{1 - a_1 \cdot z^{-1} - \cdots - a_m \cdot z^{-m}}$$

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Cookbook Process

- **Step 6 – Solve for G(z),**

$$\frac{G(z)}{F(z)} = H(z) = T_m \cdot \alpha \cdot \frac{1}{1 - e^{-\alpha \cdot T_m} \cdot z^{-1}}$$

$$G(z) = b_0 \cdot F(z) + a_1 \cdot G(z) \cdot z^{-1} \qquad a_1 = e^{-\alpha \cdot T_m} \qquad b_0 = T_m \cdot \alpha$$

- **Step 7 Inverse z transform to get back to the discrete time domain**

$$g_n = b_0 \cdot f_n + a_1 \cdot g_{n-1}$$

Done!

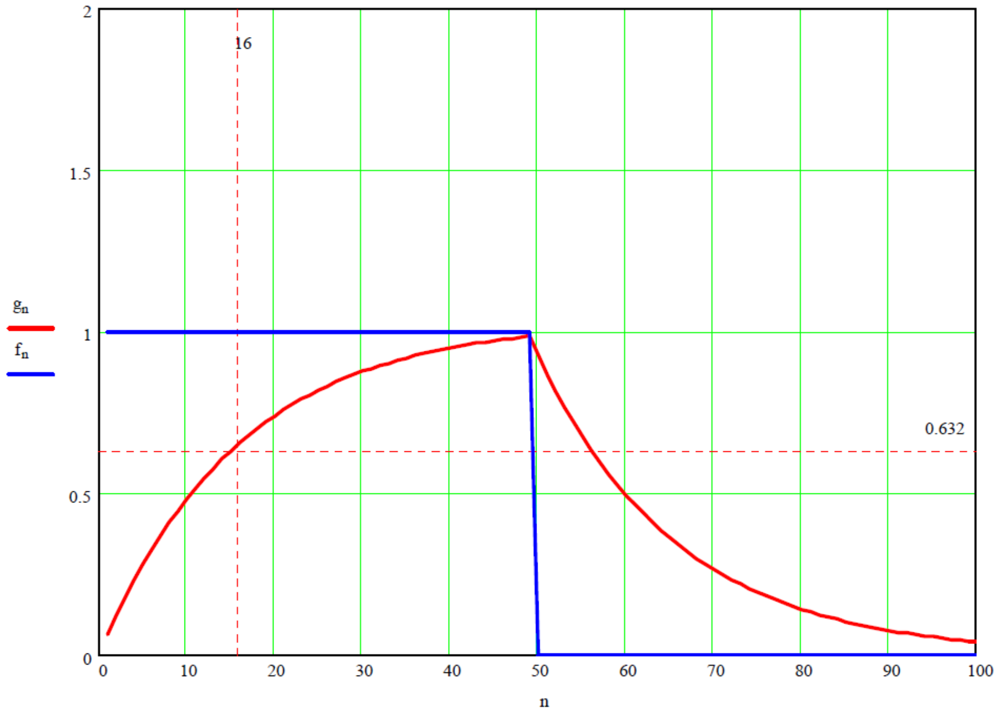
- **One more step – Check the results…**
- **Time domain**

$n = 1..100 \qquad 100 \cdot T_m = 100 \times 10^{-6}$

$f_n = u(n) - u(n-50) \qquad g_0 = 0$

$g_n = b_0 \cdot f_n - a_1 \cdot g_{n-1}$

The digital domain response is plotted.



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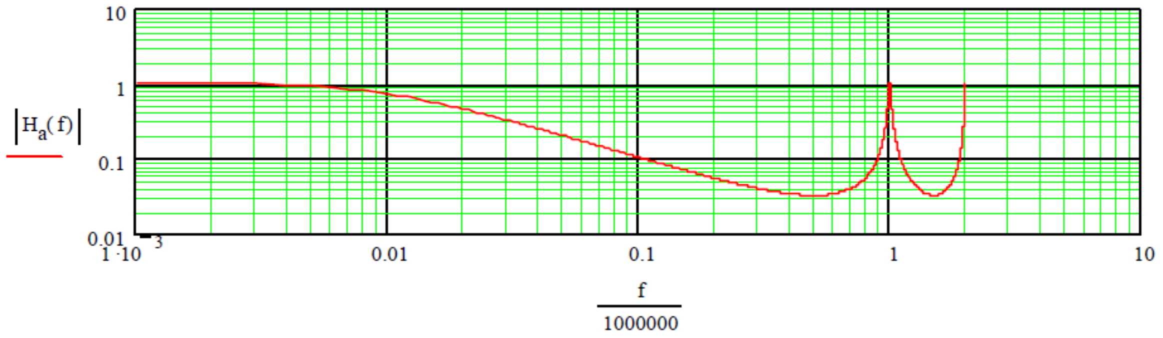
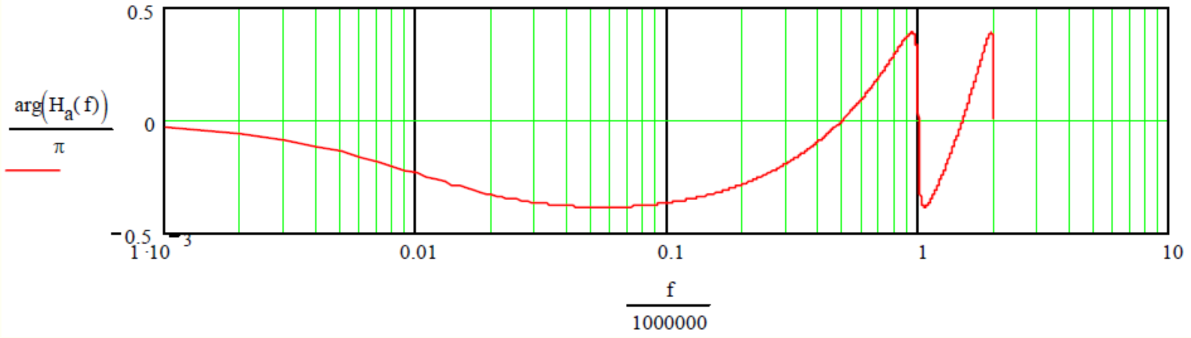
Cookbook Process

- **Frequency domain**

$$H_a(f) = H\left(e^{j\cdot 2\pi\cdot f\cdot T_m}\right)$$

$$f = \frac{f_b}{10}, \frac{f_b}{5} .. 200\cdot f_b$$





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Cookbook Process Summary

- **Summary**

Process	Description	Why / Comments
Step 1	Describe the system in the frequency domain.	Define what you would like the end result to represent. Engineers are most familiar with frequency domain representations of systems.
Step 2	Perform an inverse Fourier transform to get the system definition into the time domain.	This is required for following steps. Engineers are typically not familiar with time domain representations of systems
Step 3	Translate to the discrete time domain by inserting substitutions and scaling.	Required since digital devices can only work in the discrete time domain.
Step 4	Perform z-transform bringing problem into the discrete frequency domain (z - domain).	Simplify math.
Step 5	Perform algebra to express denominator with a leading 1, and terms of z raised to a negative integer.	Necessary for simplification.
Step 6	2nd part of algebra to solve for the desired output G(z).	Necessary for simplification.
Step 7	Simple inverse z- transform to get back to discrete time domain.	Provides the end result.
Step 8	Check for convergence and evaluate performance.	Save time by clearly understanding the results before implementing them.

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Construct Method

- **Seems time consuming – Do we need to do this every time???** **NO**
- **We are going to use the Construct Method!!!**
- **For control systems, we can make anything we need with four basic constructs**
  - **Integrator – or zero frequency pole**
  - **Differentiator – or zero frequency zero**
  - **Single pole at a break frequency**
  - **Single zero at a break frequency**
- **Details are worked out in the accompanying MathCad File.**

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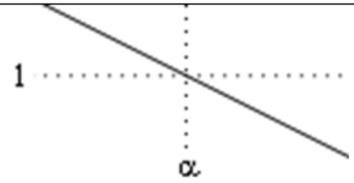
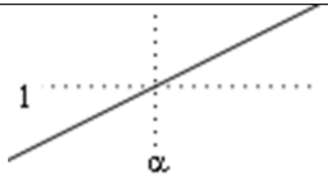
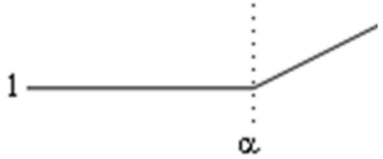
Construct Method

Construct	Description	z - domain function	Coefficients: a1	Coefficients: b0	Coefficients: b1	Frequency domain graphic
1	LPF Low Pass Filter	$H(z) = \frac{b_0}{1 - a_1 \cdot z^{-1}}$	$a_1 = e^{-\alpha \cdot T_m}$	$b_0 = \alpha \cdot T_m$	-	
2	ZFP Zero frequency pole	$H(z) = \frac{b_0}{1 - a_1 \cdot z^{-1}}$	$a_1 = 1$	$b_0 = \alpha \cdot T_m$	-	
3	ZFZ Zero frequency zero	$H(z) = b_0 + b_1 \cdot z^{-1}$	-	$b_0 = \frac{1}{\alpha \cdot T_m}$	$b_1 = \frac{-1}{\alpha \cdot T_m}$	
4	f_bZ Frequency f_b zero	$H(z) = b_0 + b_1 \cdot z^{-1}$	-	$b_0 = \frac{1}{\alpha \cdot T_m}$	$b_1 = \frac{-e^{-\alpha \cdot T_m}}{\alpha \cdot T_m}$	

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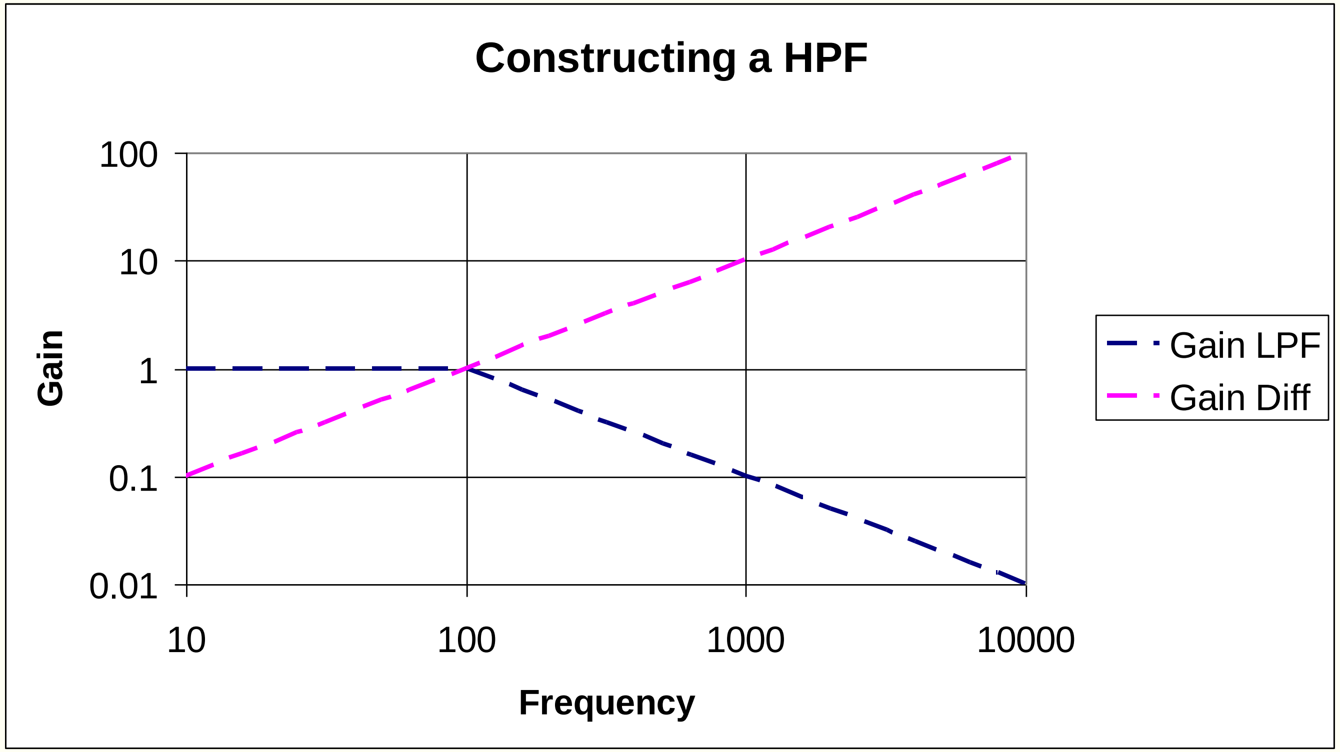
Construct Method

- **Demonstration of using constructs to build a High Pass filter (HPF).**
  - **We will use two of the four basic constructs. The Differentiator in series with a Low Pass Filter.**

  In the Frequency domain, the response of these constructs will multiply, yielding the desired result which is the High Pass Filter.



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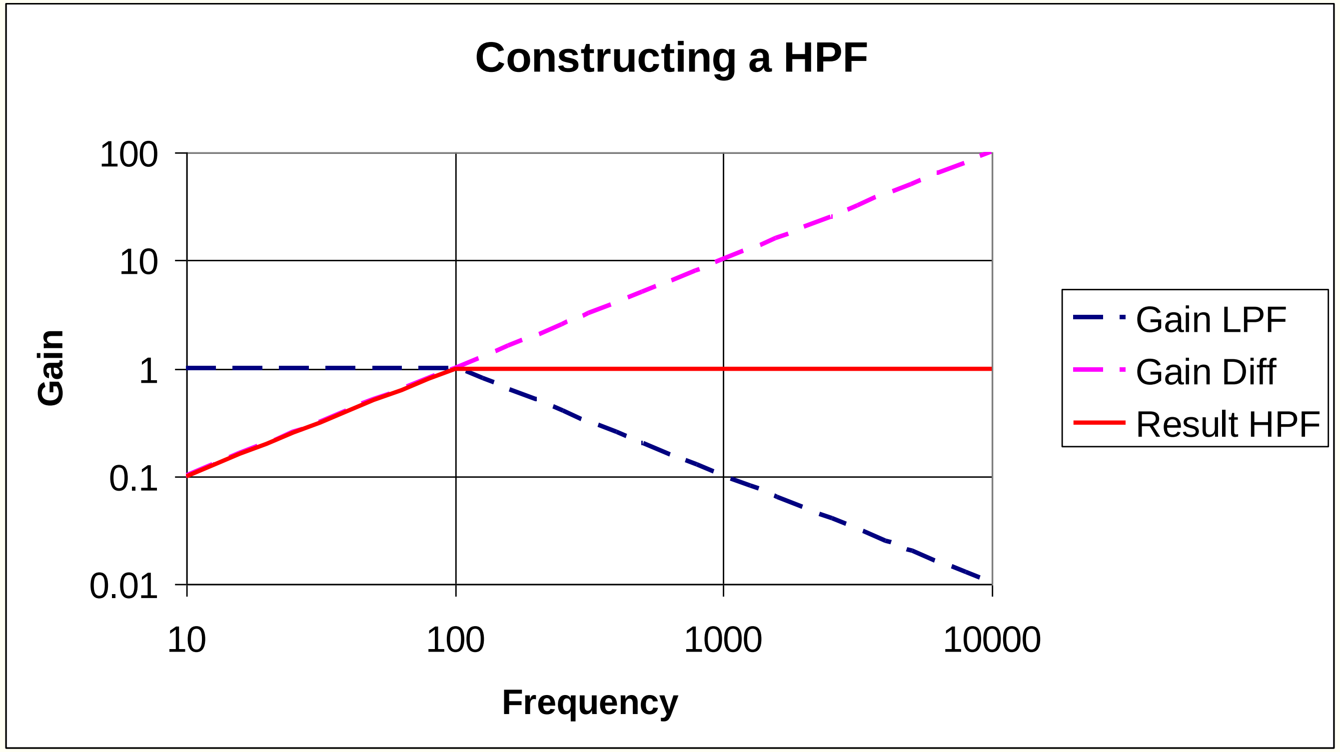
Construct Method

- **Demonstration of using constructs to build a High Pass filter (HPF).**



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Construct Method

- **Using constructs … requires only algebra.**
  - **Start by using the Z domain constructs given earlier.**

Select a breakpoint and math Frequency

$$f_b = 100 \qquad F_m = 40k \quad \text{where} \quad T_m = \frac{1}{F_m}$$

Now we can look up the constructs off the table.

$$H_{zero}(z) = \frac{1 - z^{-1}}{T_m \cdot \alpha_g} \qquad H_{LPF}(z) = T_m \cdot \alpha_g \cdot \frac{1}{1 - e^{-\alpha_g \cdot T_m} \cdot z^{-1}}$$

These are now combined by multiplying them (Simplification is done by inspection in this case).

$$H_{HPF}(z) = H_{zero}(z) \cdot H_{LPF}(z) = \frac{1 - z^{-1}}{1 - e^{-\alpha_g \cdot T_m} \cdot z^{-1}}$$

The coefficients readily apparent.

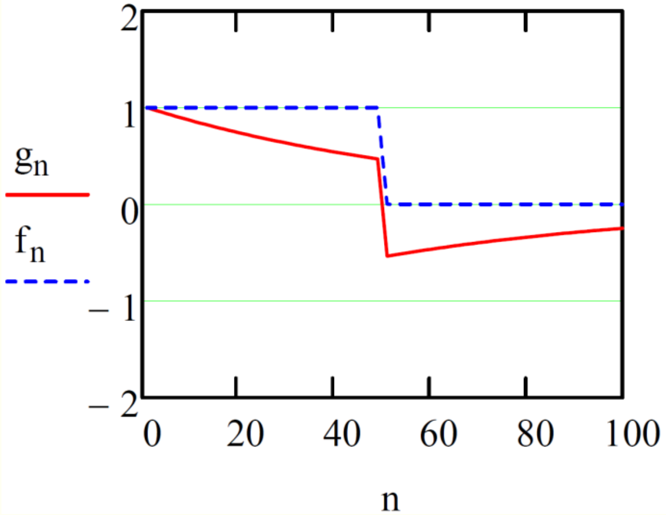
$$b_0 = 1 \quad b_1 = -1 \quad a_1 = e^{-\alpha_g \cdot T_m}$$

Done! Only using algebra.

- **Using constructs … Evaluating HPF.**
  - **Plug the previous values into the linear equation.**

$$g_n = f_n - f_{n-1} + a_1 \cdot g_{n-1}$$

  - **Choose a math program to display the time domain results graphically (MathCad chosen here).**

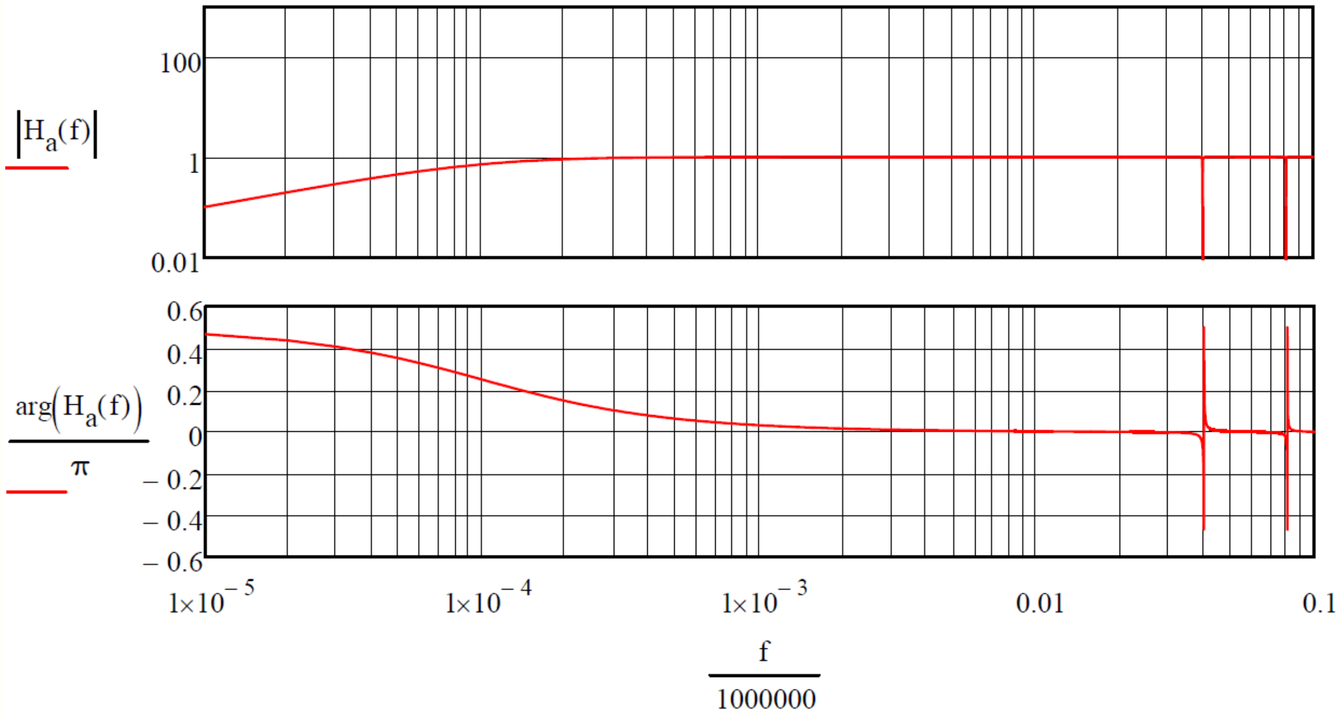


  - **This is what we should expect for a HPF. The edges come through, then the output converges towards zero.**

- **Using constructs … Evaluating HPF.**
  - **Frequency Domain evaluation.**

To translate from the z domain to the frequency domain a simple substitution for z is needed.

$$H_a(f) = H_{HPF}\left(e^{j\cdot 2\pi\cdot f\cdot T_m}\right)$$



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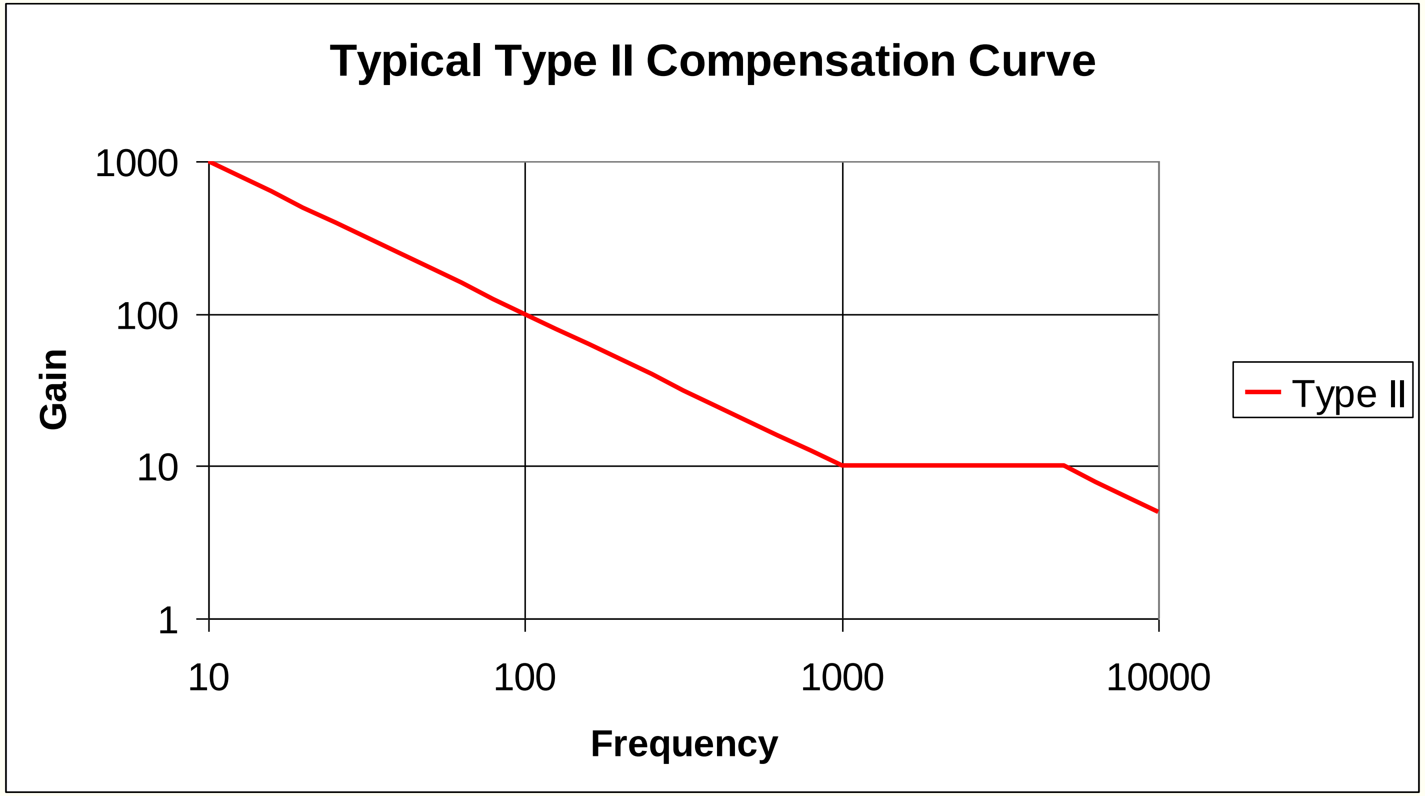
Construct Method

- **Using constructs … Building a Type II compensation network.**
  - **Common scheme used in control systems**



  - **Notice "Break" frequencies at 1000Hz and 5000Hz**

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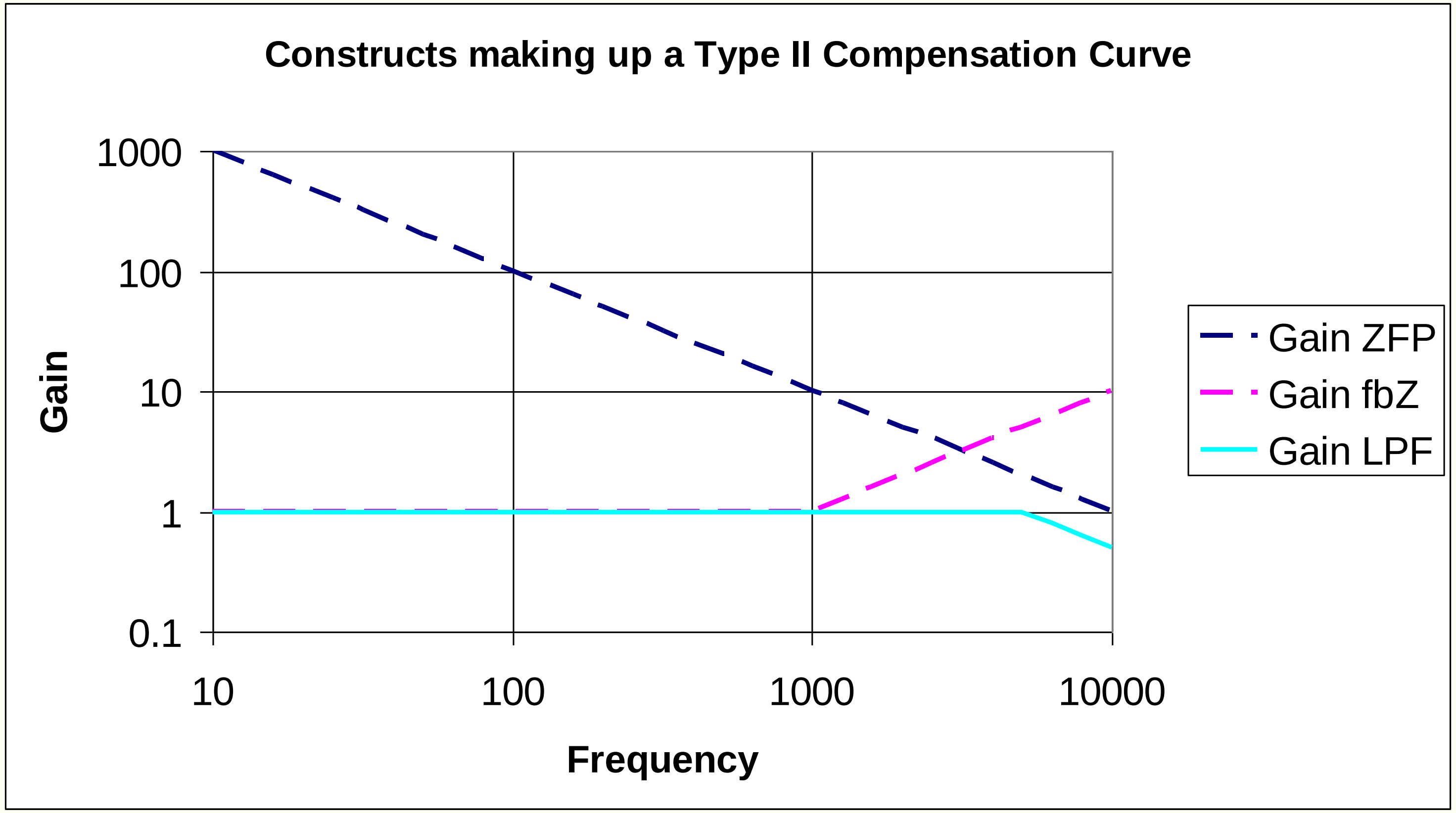
Construct Method

- **Using constructs … Type II Network**
  - **We will use 3 constructs – A Zero frequency pole, a zero at break frequency 1, and a pole at break frequency 2 (LPF).**



- **Using constructs … Type II Network**
  - **We can co right to work in the z domain, picking off construct from the table.**

First Listing Fixed Parameters

$$f_{b1} = 10k \qquad f_{b2} = 1k \qquad f_{b3} = 5k$$

$$\alpha_1 = 2\pi \cdot f_{b1} \qquad \alpha_2 = 2\pi \cdot f_{b2} \qquad \alpha_3 = 2\pi \cdot f_{b3}$$

The Math Frequency selected for this problem

$$f_m = 40k \qquad T_m = \frac{1}{f_m}$$

Pick the constructs off the Table

$$H_{ZFP}(z) = \frac{\alpha_1 \cdot T_m}{1 - z^{-1}} \qquad H_{fbZ}(z) = \frac{1}{\alpha_2 \cdot T_m} + \frac{-e^{-\alpha_2 \cdot T_m}}{(\alpha_2 \cdot T_m)} \cdot z^{-1} \qquad H_{LPF}(z) = \frac{\alpha_3 \cdot T_m}{1 - e^{-\alpha_3 \cdot T_m} \cdot z^{-1}}$$

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Construct Method

- **Using constructs … Type II Network**
  - **The Bad news … Its time for Algebra. But we will use Mathcad to do the heavy lifting.**

Poles Only

$$v_{den} := \left(1 - z^{-1}\right)\left(1 - e^{-\alpha_3 \cdot T_m} \cdot z^{-1}\right) \text{ simplify } \rightarrow \frac{\left(z - e^{-\frac{1}{4}\cdot\pi}\right)\cdot(z-1)}{z^2} \text{ coeffs } \rightarrow \begin{bmatrix} \left(e^{\pi}\right)^{\frac{-1}{4}} \\ -\dfrac{1}{\left(e^{\pi}\right)^{\frac{1}{4}}} - 1 \\ 1 \end{bmatrix}$$

Denominator From fb1

Denominator From fb3

  - **Looks Complicated, but we're just copying and pasting – MathCad is doing the work.**

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Construct Method

- **Using constructs … Type II Network**
  - **Now some minor steps to extract the "a" coefficients from MathCad.**

$$p := 0..2$$

$$a_p := \left[\left(-v_{den}\right)_{2-p}\right] \qquad a = \begin{pmatrix} -1 \\ 1.4559 \\ -0.4559 \end{pmatrix}$$

  - **Now for the Numerator – Just take the combined expression and multiply by the Denominator Term**

$$v_{num} := \underbrace{H_{T2}(z)}_{\text{Complete Term}} \cdot \left[\underbrace{\left(1 - z^{-1}\right)\cdot\left(1 - e^{-\alpha_3 \cdot T_m} \cdot z^{-1}\right)}_{\text{Denominator}}\right] \text{ simplify } \rightarrow \frac{5\cdot\pi\cdot e^{-\frac{\pi}{20}}}{2\cdot z} - \frac{5\cdot\pi}{2} \text{ coeffs } \rightarrow \begin{bmatrix} \dfrac{5\cdot\pi}{2\cdot\left(e^{\pi}\right)^{\frac{1}{20}}} \\ -\dfrac{5\cdot\pi}{2} \end{bmatrix}$$

  - **MathCad does the work**

- **Using constructs … Type II Network**
  - **Now extract the "b" coefficients from MathCad.**

$$p := 0 .. 1$$

$$b_p := \left[\left(v_{num}\right)_{1-p}\right] \qquad b = \begin{pmatrix} -7.854 \\ 6.7123 \end{pmatrix}$$

  - **Reminder – The form of the z domain equation**

$$H_{T2}(z) = \frac{b_0 + b_1 \cdot z^{-1}}{1 - a_1 \cdot z^{-1} - a_2 \cdot z^{-2}}$$

  - **In the Discrete Time Domain**

$$g_n = b_0 \cdot f_n + b_1 \cdot f_{n-1} + a_1 \cdot g_{n-1} + a_2 \cdot g_{n-2}$$

    - **Notice – 2 poles** (a_1 and a_2), 1 zero (b_1). Terms a_0 and b_0 do not create any frequency dependent terms.
    - This can be implemented in Hardware

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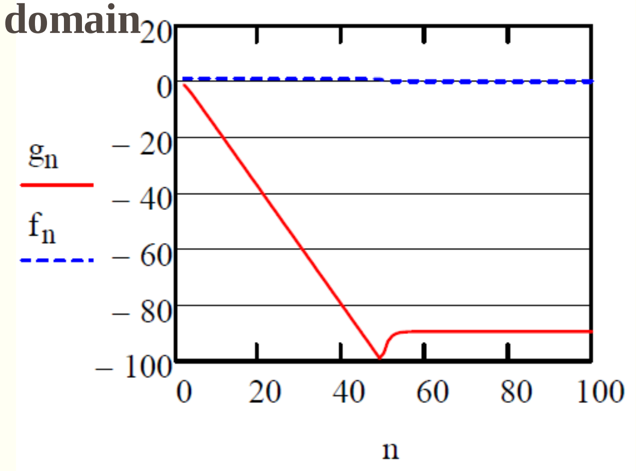
Construct Method

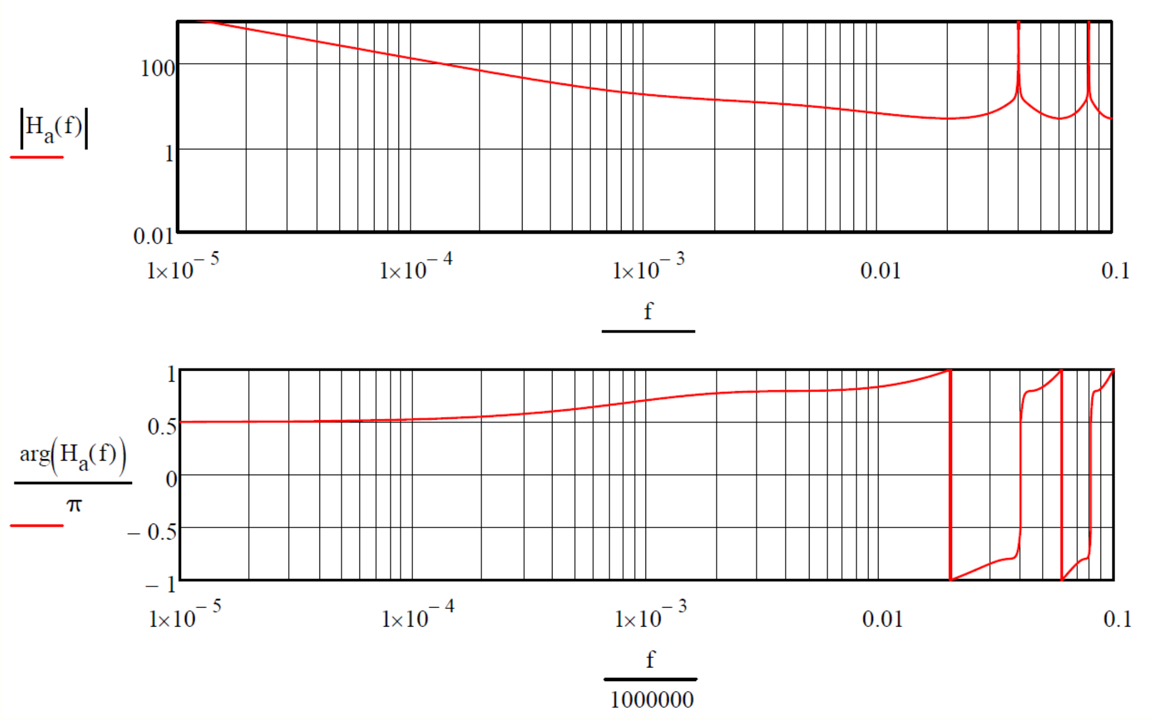
- **Using constructs … Type II Network**
  - **Check Time domain**



We followed the rules

We get convergence

  - **Check Frequency Domain**



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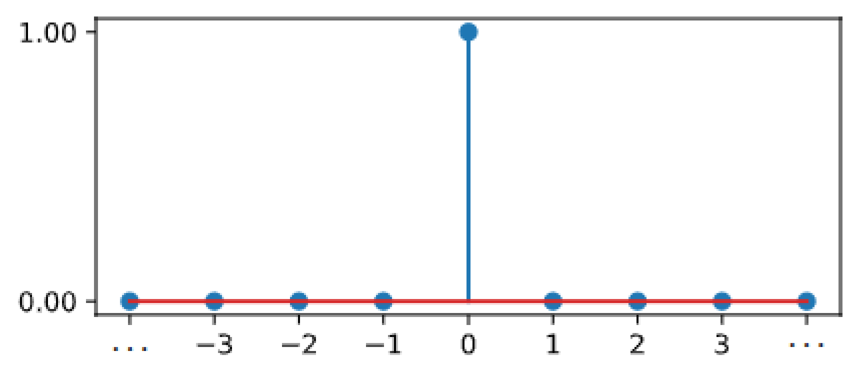
IIR vs. FIR

- **What is meant by impulse response???**
  - **If you apply an impulse to the input of a system, the impulse response will describe the gain of the system.**
  - **WHY…**

…In discrete time, an impulse, or delta function, is one for n = 0 and 0 for all other n.

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



A Primer on Digital Control for Power Converters
IIR vs. FIR

- **Performing a Discrete Fourier Transform (DFT) on an impulse…**

$$x[\alpha] = \sum_{n=-\infty}^{\infty} \delta[n]\, e^{-j\alpha n} = 1 \qquad \text{for all } \alpha$$

- **This represents a flat spectrum with amplitude of 1 and zero phase shift for all frequencies.**
- **Therefore – if a time domain impulse is applied to the input, the response (in the frequency domain) will be the gain of the system.**

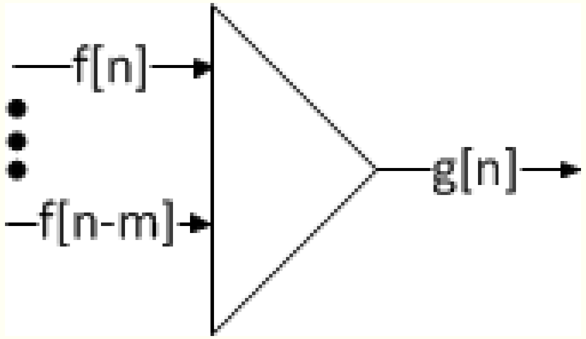
A Primer on Digital Control for Power Converters
IIR vs. FIR

- **What do the terms Finite and Infinite refer to?...The terms Finite and Infinite refer to quantity of time.**

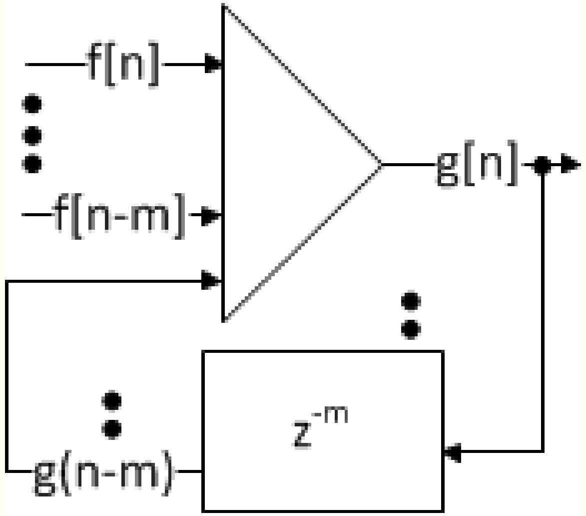
- **A FIR uses only present and past values of input to produce the output. It does not have memory of its current state. When the input ceases, the output will be finite depending on the number of terms or registers used to construct the filter.**

- **A IIR uses present and past values of input and past values of the output. This system has memory of its past states. The output will continue forever since past values of output are used as input.**

FIR



IIR



A Primer on Digital Control for Power Converters
IIR vs. FIR

- **An example of a FIR is a moving average filter. A finite number of past inputs are given equal value.**

$$g_{FIR_n} = 0.1\cdot\left(f_n + f_{n-1} + f_{n-2} + f_{n-3} + f_{n-4} + f_{n-5} + f_{n-6} + f_{n-7} + f_{n-8} + f_{n-9}\right)$$

- **An arguably similar IIR filter can be a low pass filter where the incoming data has a similar weight, represented as follows.**

$$g_{IIR_n} = 0.1 f_n + 0.9 g_{IIR_{n-1}}$$

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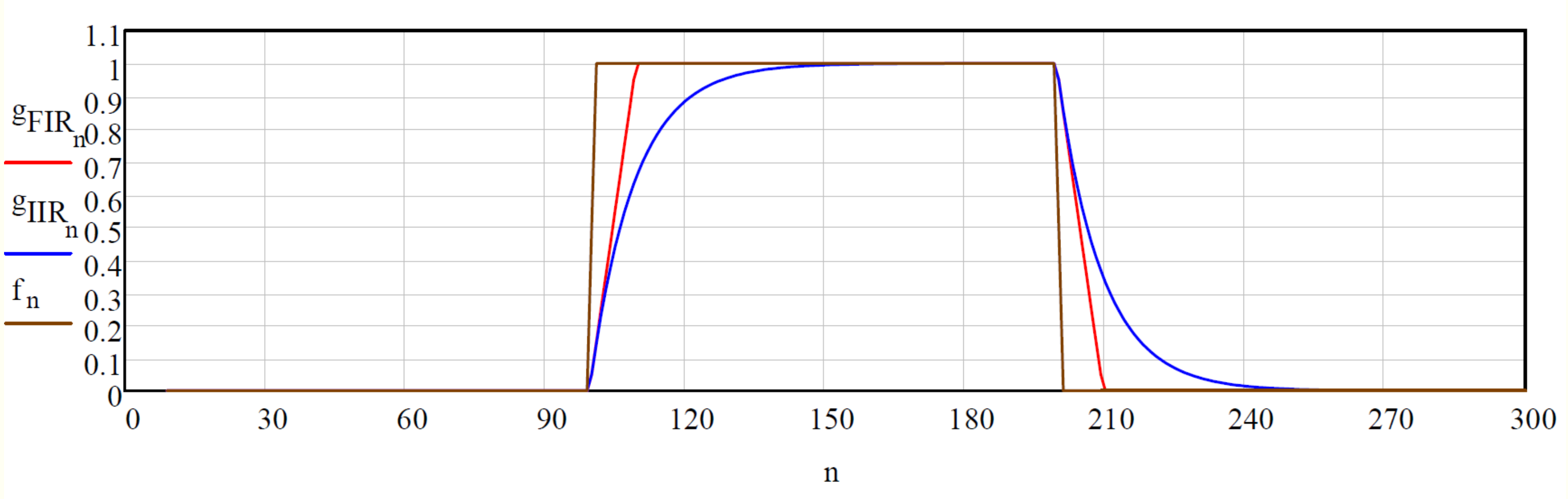
IIR vs. FIR

- **The similar IIR filter will use a portion of the previous output, and a weighted sample of an incoming data stream.**



- **When stimulated with a step function, they start out with a similar slope, however they diverge after 3 or 4 samples. After 10 samples the FIR is completely settled while the IIR continues to exponentially settle.**

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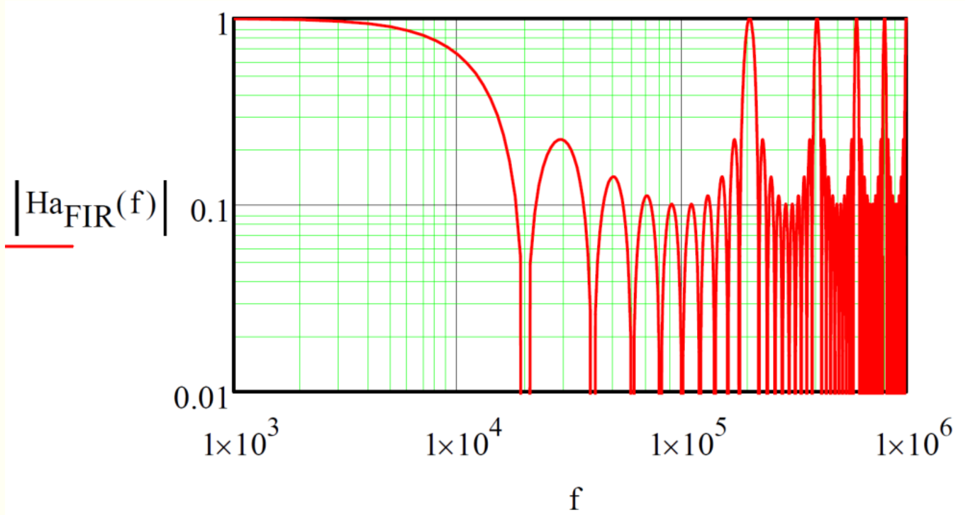
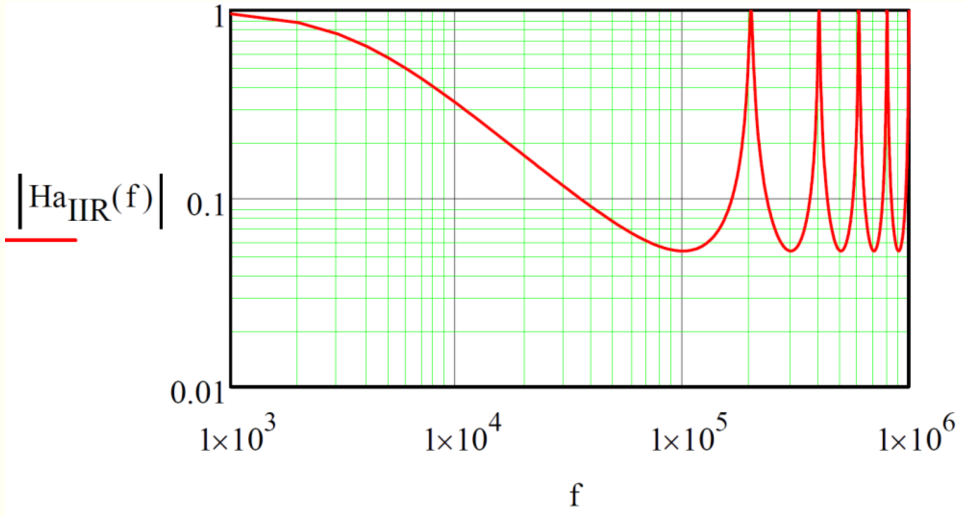
IIR vs. FIR

- **Frequency Domain Representation – Sample Rate set to 200 kHz**
  - **Aliasing is significant at ½ of the Sample rate.**
  - **Peaks demonstrate the need for Anti-aliasing filters**

$$Ha_{FIR}(f) = H_{FIR}\left(e^{j\cdot 2\pi f\cdot Hz\cdot T_m}\right)$$

$$Ha_{IIR}(f) = H_{IIR}\left(e^{j\cdot 2\pi f\cdot Hz\cdot T_m}\right)$$





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IIR vs. FIR

- **IIR Attributes**
  - **Less terms = higher speed and fewer resources. Better choice for embedded real time processing.**
  - **Works well for small signal and large signal output variations. Better match replacing analog control loops.**
  - **Narrowing the scope to control loops, designs are generally less complex.**
  - **Feedback term leaves a potential for instability.**
- **FIR Attributes**
  - **Works well in environments where resources are abundant and execution speed is not critical, such as post processing of data on a host PC.**
  - **No feedback means the filter is unconditionally stable. It can not oscillate, and it will always converge.**
  - **Many taps are required to span a wide frequency range.**
  - **There are several websites and applications to assist in complex designs.**

A Primer on Digital Control for Power Converters
Short-Cut to LPF

- **Looking back at the frequency response curve for the last IIR, By inspection, the cutoff frequency (-3db point) appears to be 3 kHz. To calculate the cutoff frequency we will use…**

$$a_1 = e^{-\alpha \cdot T_m} = 0.9$$

where $\alpha = 2\pi f_c$

Solving for Frequency $f_c = 3.35\text{kHz}$

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Short-Cut to LPF

- **Working Forward this time, Let's construct an LPF with a 10kHz breakpoint in a system with 200 kHz sample rate.**
- **The Linear Form will be…** $g_n = b_0 \cdot f_n + a_1 \cdot g_{n-1}$ **(we like this form since we can easily implement it in HW or FW)**

Letting $\alpha = 2\pi \cdot f_c = 62.832 \frac{\text{kRad}}{\text{sec}}$

- **The Output will decay by** $e^{-\alpha \cdot T_m}$ **for each sample period.**
  - **Consider the case where the input f_n ceases, it should be obvious that the decay coefficient is a_1.**
  - **Next Consider the case where f_n is stable for a long time. Here $b_0 = (1 – a_1)$.**

$$a_1 = e^{-\alpha \cdot T_m} = 0.73 \qquad b_0 = 1 - a_1 = 0.27$$

A Primer on Digital Control for Power Converters
Convergence

- **Does the Construct Method Guarantee Convergence?**

$$g_n = f_n b_0 + g_{n-1} a_1$$

Looking at the discrete time domain equation above, stable results will be provided when $0 \le a_1 \le 1$.

This is seen in how the equation responds to an impulse and how it responds to a sustained value.

After an impulse, g_n will take on a value and f_n will be zero.

If $a_1 = 0$	g_n will become zero instantly.	✔
If $a_1 = 1$	g_n will stay at g_{n-1} indefinitely.	✔
If $0 < a_1 < 1$	g_n will decay exponentially towards zero.	✔
If $1 < a_1$	g_n will increase indefinitely.	✖
If $-1 < a_1 < 0$	g_n will decay, but oscillate between positive and negative values.	✖
If $a_1 = -1$	g_n will oscillate between $-g_{n-1}$ and g_{n-1}.	✖
If $a_1 < -1$	$\lvert g_n \rvert$ will increase indefinitely.	✖

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Convergence

- **Conclusions on Convergence**

For a converging LPF Construct, an impulse input will converge to zero and be stable. ✔

For any sustained input, the LPF will converge to that input value times b_0. ✔

For a converging Integrator Construct, an impulse settles to a fixed value and holds it. ✔

A sustained value will cause the integrator to increase indefinitely. ✖

Therefore, any combination of constructs will always converge as long as only ONE integrator is used.

A Primer on Digital Control for Power Converters
Introducing Gain On the Fly

- **How do we introduce Gain???**
- **In the equation, $g_n = b_0 \cdot f_n + a_1 \cdot g_{n-1}$, it seems intuitive just to multiply b_0 and a_1 by some factor to introduce gain.**
- **This would be wrong**
  - **g_{n-1} would already include the multiplication factor, multiplying a_1 would be repetitive.**
  - **Multiplying a_1 may not follow the bounds rules discussed previously.**
- **This statement becomes more obvious in the Z domain.**

$$G(z) = b_0 F(z) + a_1 G(z) \cdot z^{-1}$$

$$H(z) = \frac{G(z)}{F(z)} = \frac{b_0}{1 - a_1 z^{-1}}$$

Multiplying b_0 by some factor will introduce gain.

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Introducing Gain On the Fly

- **Live Tour of Excel and Mathcad sheets to share.**

- **Live Tour of a Mathcad file used for Bode analysis using Standard Lab Equipment**